

Heavy neutrino search in accelerator-based experiments

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Abstract

We explore the feasibility of detecting heavy neutrinos by the existing facilities of neutrino experiments. A heavy neutrino in the mass range $1 \text{ MeV} \lesssim M_N \lesssim 500 \text{ MeV}$ is produced by pion or kaon decay, and decays to charged particles which leave signals in neutrino detectors. Taking the T2K experiment as a typical example, we estimate the heavy neutrino flux produced in the neutrino beam line. Due to massive nature of the heavy neutrino, the spectrum of the heavy neutrino is significantly different from that of the ordinary neutrinos. While the ordinary neutrinos are emitted to various directions in the laboratory frame due to their tiny masses, the heavy neutrinos tend to be emitted to the forward directions and frequently hit the detector. The sensitivity for the mixing parameters is studied by evaluating the number of signal events in the near detector ND280. The sensitivity of T2K at 10^{21} POT is found to be comparable to that of the previous experiment PS191, which has placed the most stringent bounds on the mixing parameters of the heavy neutrinos for $140 \text{ MeV} \lesssim M_N \lesssim 500 \text{ MeV}$.

1 Introduction

In the last few decades, neutrino oscillation experiments have conclusively shown that neutrinos are massive [1]. The minimal version of the Standard Model is thus to be extended to accommodate the neutrino masses. In many possible extension of the Standard Model, neutral heavy leptons are often predicted. In the seesaw mechanism [2] for example, the right-handed neutrinos are introduced and they weakly mix with the ordinary neutrinos after the electroweak symmetry breaking.

For the mass of the heavy neutrinos, a wide range of possibilities has been discussed in literature. In the canonical picture of the seesaw mechanism, heavy neutrino masses are supposed to be around the grand unification scale. These super heavy neutrinos can account for the baryon asymmetry of the universe by the leptogenesis scenario [3]. Another possibility to account for the baryon asymmetry has been suggested in [4, 5] and further studied in [6]. In this scenario, two quasi-degenerate heavy neutrinos of $\mathcal{O}(100)$ MeV – $\mathcal{O}(10)$ GeV play a crucial role in the early universe. Heavy neutrinos in the mass range ~ 0.2 GeV could enhance the energy transport from the core to the stalled shock and favor the supernova explosion [7]. Heavy neutrinos with a few keV mass have also attracted much interests as a viable dark matter candidate [8] and an agent of the pulsar velocities [9]. Remarkably, the dark matter and the baryon asymmetry due to keV and GeV heavy neutrinos can originate in a simple framework so called ν MSM [10, 5], which is an extension of the Standard Model with just three generations of the right-handed neutrinos. Besides the super heavy range much larger than TeV, such heavy neutrinos can be tested in existing and forthcoming experiments due to lower threshold energies of production (for example, see Ref. [11, 12] and references therein).

In this paper, we focus on the heavy neutrinos produced by kaon decays. The previous neutrino experiments, including peak searches in the meson decays [13, 14, 15, 16, 17] and the decay search with accelerators [18, 19], have placed stringent bounds on the mixing parameters in this mass range. In particular, PS191 [19] has placed the strongest bounds for the mixing parameters in the mass range of $140 \text{ MeV} \lesssim M_N \lesssim 500 \text{ MeV}$. Since the PS191 experiment in 1984, however, no further experiments of this type of decay search have performed and the bounds have not been updated for about 30 years. On the other hand, great progress has been made in neutrino oscillation experiments over the same period. It is interesting to note that typical long or short baseline experiments are equipped with the (near) detectors placed at $\mathcal{O}(100)$ meters away from the beam targets,

	PS191 [19]	T2K [23]	MINOS [24]	MiniBooNE [25]	SciBooNE [26]
POT	0.86×10^{19}	10^{21}	10^{21}	10^{21}	10^{21}
(Distance) $^{-2}$	$(128 \text{ m})^{-2}$	$(280 \text{ m})^{-2}$	$(1 \text{ km})^{-2}$	$(541 \text{ m})^{-2}$	$(100 \text{ m})^{-2}$
Volume	216 m^3	88 m^3	303 m^3	524 m^3	15.3 m^3
Events	1	9.9	2.7	15.8	13.5

Table 1: A comparison between PS191 and recent accelerator experiments. The item “Distance” means the distance between the beam target and the detector for each experiment. The item “Events” shows $\text{POT} \times (\text{Distance})^{-2} \times \text{Volume}$ in units of PS191. The POTs for the oscillation experiments are assumed to achieve 10^{21} .

which detectors are capable of measuring the charged-particle tracks produced by the heavy neutrino decays. A natural question is then whether the existing accelerator-based neutrino experiments are capable of discovering the heavy neutrinos and how sensitive such experiments are*. We believe that this is a timely question to ask. In fact, the exposure of PS191 is about $200 \text{ m}^3 \times 10^{20}$ POT while the recent accelerator-based neutrino experiments are expected to achieve 10^{21} POT with the near detectors which are typically no smaller than 200 m^3 by factor of 10. Table 1 shows a comparison between PS191 and several examples of recent neutrino experiments. Among several options of accelerator experiments, in this paper we focus on the T2K experiment as a typical example.

The study follows two main steps; the flux estimation and the event number calculation for various signal decays. First, in the flux estimation, we use a semi-analytical method with a help of the active neutrino flux ϕ_ν simulated by the T2K collaboration [27, 28, 29], making a reasonable simplification for the geometry of the decay tunnel and the detector. More precise fluxes of the heavy neutrino might be obtained by Monte Carlo methods. In a Monte Carlo calculation it is possible to take into account the details of the geometry and the spectrum of the parent mesons. The analytical technique is nevertheless useful, since it allows one to understand the essential physics which determines the behavior of the heavy neutrino spectrum, in particular its mass dependence. We emphasize that the phase-space effect in kaon decay is important and the heavy neutrino flux ϕ_N can be significantly deviated from the naive expectation $\phi_N \simeq |\Theta|^2 \phi_\nu$, where Θ is the active-heavy mixing parameter of interest. As the heavy neutrino mass M_N approaches to the production threshold, the heavy neutrinos tend to be distributed to lower energies with a narrow spread, so that the proportionality $\phi_N \simeq |\Theta|^2 \phi_\nu$ is broken. This phase-space effect

*Far detectors with large volume are also useful to detect the heavy neutrinos produced in the atmosphere [20, 21].

leads to larger event numbers than the naive expectation and has a significant impact for the estimation of the sensitivity.

Second, in the event number calculation, we take into account various decay modes of the heavy neutrino N . The two-body modes $N \rightarrow e^\mp \pi^\pm$ and $N \rightarrow \mu^\mp \pi^\pm$ are the most promising channels due to their large branching ratios. For these modes, the invariant mass distribution for the lepton and the pion momenta has a peak at the heavy neutrino mass. The three-body modes $N \rightarrow e^- e^+ \nu$, $N \rightarrow \mu^\mp e^\pm \nu$ and $N \rightarrow \mu^- \mu^+ \nu$ are also interesting. While the first mode may suffer higher background by π^0 decay, the latter two modes have smaller backgrounds and also serve as promising channels for discovering the heavy neutrino. By assuming non-observation of these exotic events in the T2K near detector at 10^{21} POT, one finds the upper-bounds for the mixing parameters comparable to PS191. This means that T2K may have a good chance to discover the heavy neutrino in near future.

The layout of this paper is as follows. In Section 2, we introduce the heavy neutrino and briefly review its essential properties. In Section 3, the flux of the heavy neutrino at the near detector is discussed. In Section 4, the decays of the heavy neutrino at the detector and their event numbers are studied. Section 5 is devoted to conclusions.

2 Properties of the heavy neutrino

We consider a heavy (sterile) neutrino N with the mass range $1 \text{ MeV} \lesssim M_N \lesssim 500 \text{ MeV}$. The flavor eigenstates of the left-handed neutrinos ν_α ($\alpha = e, \mu, \tau$) are given by the linear combination of the mass eigenstates as

$$\nu_\alpha = U_{\alpha i} \nu_i + \Theta_\alpha N, \quad (1)$$

where $U_{\alpha i}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, ν_i ($i = 1, 2, 3$) are the mass eigenstates of the active neutrinos. The parameter Θ_α is the mixing between (light) active and sterile neutrinos, through which N interacts with the weak gauge bosons. The extension to the multi-generation case is trivially done by replacing $\Theta_\alpha N$ with $\sum_I \Theta_{\alpha I} N_I$. In this paper, we assume N is Dirac particle unless otherwise stated. This is simply because we would like to make a comparison to PS191 in which the same assumption is made. If N is Majorana particle, the decay width is doubled since N decays to charge-conjugated states also.

With the mass range of $140 \text{ MeV} \lesssim M_N \lesssim 500 \text{ MeV}$, the heavy neutrino N is produced by K decay. The main production modes are

$$K^+ \rightarrow \mu^+ N, \quad K^+ \rightarrow e^+ N \quad (2)$$

for $M_N < 388 \text{ MeV}$ and $M_N < 493 \text{ MeV}$ respectively. The decay width $K^+ \rightarrow \mu^+ N$ ($K^+ \rightarrow e^+ N$) is proportional to $|\Theta_\mu|^2$ ($|\Theta_e|^2$), and Θ_τ is irrelevant for the production[†]. We describe further details of the decay modes (2) in Section 3 and Appendix A. Since the magnetic horn focuses positively-charged mesons, the contributions from K^- are small [28] and in what follows we neglect the K^- contributions.

The heavy neutrinos N produced by the meson decays escape from the decay volume and part of them are injected into the near detector ND280 in the T2K experiment and decay to leave signals. Depending on the mass, N decays to lighter particles in various decay modes;

$$\begin{aligned} N &\rightarrow \gamma\nu, \quad N \rightarrow 3\nu, \quad N \rightarrow e^- e^+ \nu, \quad N \rightarrow \mu^\mp e^\pm \nu, \\ N &\rightarrow \nu\pi^0, \quad N \rightarrow e^- \pi^+, \quad N \rightarrow \mu^- \mu^+ \nu, \quad N \rightarrow \mu^- \pi^+. \end{aligned}$$

With the ND280 detector capable of identifying e^\pm, μ^\pm and π^\pm , some of the above channels can be detected.

Due to large branching ratios, the two body modes $N \rightarrow \nu\pi^0$, $N \rightarrow e^- \pi^+$ and $N \rightarrow \mu^- \pi^+$ would be the most frequent events. However $N \rightarrow \nu\pi^0$ does not seem to be as promising as the other two modes since π^0 are copiously produced by the ordinary neutrino interactions and may suffer from large backgrounds. On the other hand, $N \rightarrow e^- \pi^+$ and $N \rightarrow \mu^- \pi^+$ leave two charged-particle tracks with monochromatic energies in the rest frame of N . The invariant mass distribution of the two charged-particle momenta sharply peaks at the heavy neutrino mass, which seems to be easier to see than a slight excess of π^0 events over huge background. We thus proceed without any further analysis of $N \rightarrow \nu\pi^0$, but study $N \rightarrow e^- \pi^+$ and $N \rightarrow \mu^- \pi^+$ in more detail in Section 4. The decay rates of $N \rightarrow e^- \pi^+$ and $N \rightarrow \mu^- \pi^+$ are proportional to $|\Theta_e|^2$ and $|\Theta_\mu|^2$ respectively. The radiative decay $N \rightarrow \gamma\nu$ is induced at one loop and negligible in this work.

In spite of small branching ratios, the three body modes $N \rightarrow e^- e^+ \nu$, $N \rightarrow \mu^\mp e^\pm \nu$ and $N \rightarrow \mu^- \mu^+ \nu$ are also interesting to study. Especially $N \rightarrow \mu^\mp e^\pm \nu$ and $N \rightarrow \mu^- \mu^+ \nu$ have smaller background than $N \rightarrow e^- e^+ \nu$ and a few these events may lead to the discovery of

[†]The heavier mesons such as D can decay producing tau so that Θ_τ is involved in the production. In this work we neglect the contribution from such heavier mesons.

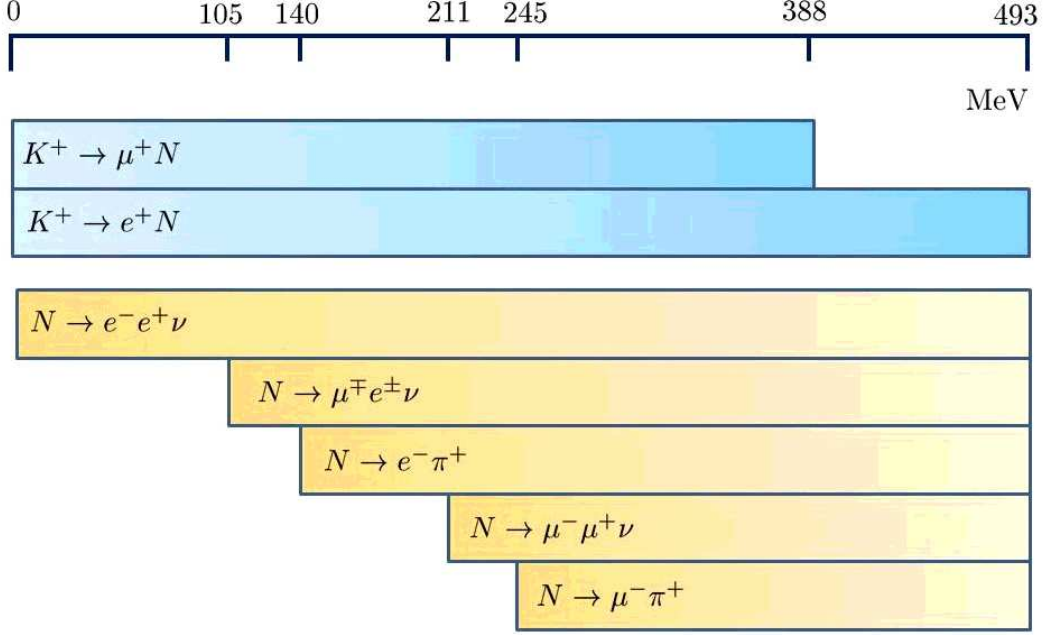


Figure 1: Summary of the production and the detection processes of the heavy neutrino N . The decay mode $N \rightarrow \nu\pi^0$ is abbreviated here because it is not as promising detection channel as the other ones.

the heavy neutrino. Notice that $N \rightarrow e^-e^+\nu$ and $N \rightarrow \mu^-\mu^+\nu$ are conducted not only by the charged-current (CC) but also by the neutral current (NC), so that these decay rates depend on all types of the mixing parameter $\Theta_{e,\mu,\tau}$. On the other hand $N \rightarrow \mu^\mp e^\pm \nu$ are mediated only by the CC interactions and the decay rate depends on $\Theta_{e,\mu}$.

Fig. 1 summarizes the production and the decay processes of N to be studied in this work. We do not present the formulas for the decay width here. A complete list of the decay modes and the decay width are found in Ref. [11].

3 Heavy neutrino flux at the near detector

In the T2K experiment, pions and kaons are produced by the interaction of 31 GeV protons with the graphite target. The produced mesons are focused by the magnetic horns and enter the decay volume of 96 m long filled with helium gas. The parent mesons decay in flight inside the decay volume. The off-axis detector ND280 is located 280 m from the target station. The off-axis angle to ND280 from the target position is 2.04° . The layout of the secondary beam line and the near detector is sketched in Fig. 2. Details of the experiment setup are found in Ref. [29].

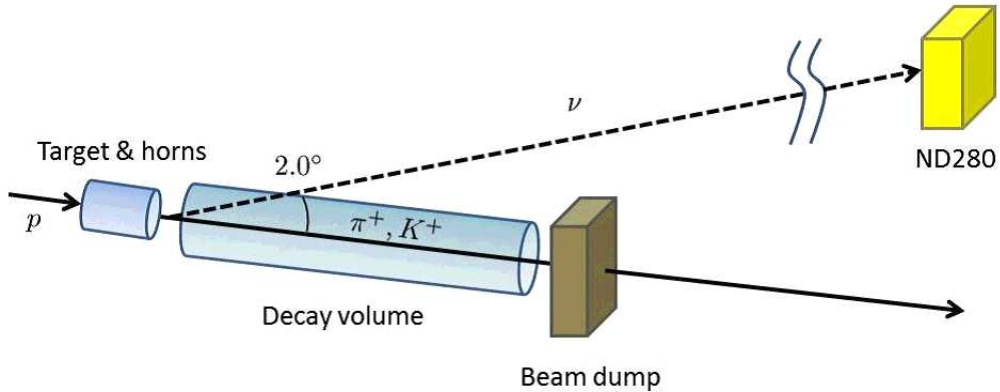


Figure 2: Schematic of the secondary beam line and the near detector ND280.

Calculation of the neutrino flux at the near detector is a complicated task. The T2K collaboration has simulated the fluxes of the active neutrinos by the Monte-Carlo method. In this work, we do not follow their approach, but estimate the heavy neutrino flux by a semi-analytical method similar to Ref. [22]. We try to reconstruct a reasonable flux of parent particles from the active neutrino flux of Ref. [28], and then evaluate the heavy neutrino flux from the reconstructed parent flux. In this paper, we focus on K^+ meson as the parent of the heavy neutrinos since K^+ decay covers wider range of the heavy neutrino mass than π^+ decay. The following discussion is however easily extended to the π^+ case.

3.1 Modeling the parent flux

For ND280, the neutrino source is a line-like object rather than a point. Let $\phi(p_K, l)$ denotes the K^+ spectrum along the decay volume, where p_K is the magnitude of the K^+ momentum and l is the flight length of K^+ . We set $l = 0$ to be the upstream end of the decay volume. In the decay volume filled with helium gas, the decay length of K^+ is much shorter than the interaction length. One thus finds

$$\phi_K(p_K, l) = \phi_K(p_K) e^{-\frac{l}{\Lambda_K}}, \quad (3)$$

where $\phi_K(p_K)$ is the spectrum at $l = 0$ and Λ_K is the total decay length $\Lambda_K = 3.7(p_K/m_K)$ m with the kaon mass $m_K = 493$ MeV.

If the parent spectrum $\phi_K(p_K)$ is known, one can calculate the daughter ν_μ flux from

$K^+ \rightarrow \mu^+ \nu_\mu$ decay. The source term of ν_μ is given by

$$\begin{aligned} S_\nu(E_\nu, \theta, \phi, l) &= \int_0^\infty dp_K \frac{\phi_K(p_K, l)}{\beta(1/\Gamma)\gamma} \frac{1}{\Gamma} \frac{d^3\Gamma}{dE_\nu d\cos\theta d\phi} \\ &= \int_0^\infty dp_K \phi_K(p_K, l) \left(\frac{m_K}{p_K} \right) \frac{d^3\Gamma}{dE_\nu d\cos\theta d\phi}, \end{aligned} \quad (4)$$

where E_ν is ν_μ energy, θ and ϕ are the polar and the azimuth angles of the emitted ν_μ relative to the K^+ momentum directions, and Γ is the $K^+ \rightarrow \mu^+ \nu_\mu$ decay width. Giving the source term, the ν_μ flux $\phi_{\nu_\mu}(E_\nu)$ at ND280 is obtained by integrating the source term along l and the angles θ and ϕ covered by ND280. It reads

$$\phi_{\nu_\mu}(E_\nu) = \int_0^{l_f} dl \int_{-1}^1 d\cos\theta \int_0^{2\pi} d\phi \frac{1}{A} S_\nu(E_\nu, \theta, \phi, l) P(\theta, \phi), \quad (5)$$

where $l_f = 96$ m, A is the effective area of ND280, $P(\theta, \phi)$ is the “projection” function which turns unity if ν_μ enters ND280 and otherwise zero.

The parent kaons carry two more degrees of freedom which are not explicitly mentioned above; the polar angle relative to the beam axis (the central axis of the decay volume) and the radial coordinate from the beam axis, both of them are defined at $l = 0$. The location of ND280 relative to each K^+ momentum depends on these degrees of freedom and the function $P(\theta, \phi)$ is in fact highly complicated. However the situation is greatly simplified if kaon momenta are assumed to be parallel to the beam axis. Information about the polar angle and the radial coordinate is then to be represented by an effective off-axis angle θ_0 , which is an “virtual” off-axis angle to ND280 from the upstream end of the decay volume. This angle θ_0 represents the average off-axis angle to ND280 from each K^+ momentum, which angle varies kaon by kaon carrying deferent polar angles and radial coordinates. Furthermore we neglect θ dependence of the effective area A . Adopting these simple modeling of geometry of the decay volume and the detector, we use

$$\phi_{\nu_\mu}(E_\nu) = \frac{\Delta\phi}{A} \int_0^{l_f} dl \int_{-1}^1 d\cos\theta S_\nu(E_\nu, \theta, l) P'(\theta, \theta_0), \quad (6)$$

as a formula to relate the daughter ν_μ flux with the parent K^+ spectrum. Here $\Delta\phi$ is ϕ interval defined by the detector width, $P'(\theta, \theta_0)$ denotes the projection function determined by the detector hight and K^+ decay point l . Since the explicite expression of Eq. (6) is rather lengthy, we put details on Eq. (6) in Appendix A .

Having Eq. (6), our strategy is fitting $\phi_{\nu_\mu}(E_\nu)$ calculated in Ref. [28] by adjusting the parameters included in the right-handed side of Eq. (6), with a kaon spectrum which

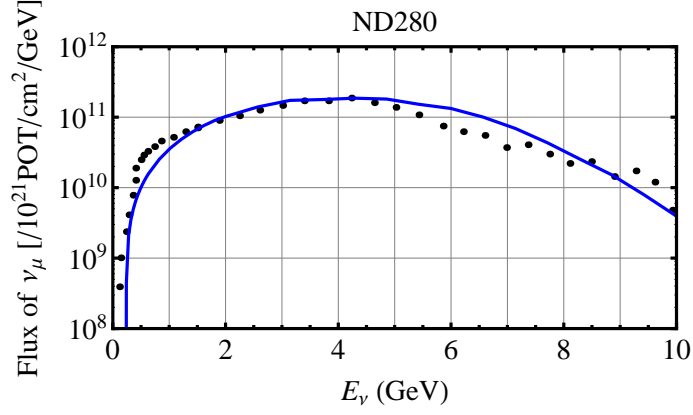


Figure 3: A comparison between Eq. (6) and the ν_μ flux simulated in Ref. [28]. Dots shows the result of the simulation in Ref. [28] and solid curve is Eq. (6) with the parameters $\theta_0 = 1.48^\circ$, $a_0 = 4.8 \times 10^{19} \text{ mb}^{-1}$ and $p_0 = 2.1 \text{ GeV}$.

is physically well-motivated. The K^+ spectrum $d\sigma/dp_K$ in the proton collision with a graphite target is measured by NA61/SHINE Collaboration [30], which is customized to improve the flux calculation in T2K. It seems reasonable to expect that the shape of the kaon spectrum is not far from this measured spectrum. However, the effects such as the secondary protons [28] and the magnetic horns must deform $d\sigma/dp_K$ to some extent. To take into account this deformation, we allow a shift of the peak of $d\sigma/dp_K$. In summary, in order to model the kaon spectrum $\phi_K(p_K)$, we introduce two free parameter a_0 and p_0 , the overall scale factor and the shift of the peak, respectively. See Appendix A for more details.

With the above ansatz for the kaon spectrum, we have three parameters in the right-hand side of Eq. (6); the effective off-axis angle θ_0 , the scale factor a_0 and the shift of the peak p_0 . Fig. 3 shows the fit by Eq. (6). The dots show the result of the simulation in Ref. [28], while the solid curve shows Eq. (6) with the parameter set of $\theta_0 = 1.48^\circ$, $a_0 = 4.8 \times 10^{19} \text{ mb}^{-1}$ and $p_0 = 2.1 \text{ GeV}$. It is seen that Eq. (6) is well tracing the global behavior of the simulated result of Ref. [28].

Notice that the effective off-axis angle is taken as $\theta_0 = 1.48^\circ$, which is smaller than the actual one 2.04° . If this angle is taken as $\theta_0 = 2.04^\circ$, the flux starts to fall off at $E_\nu \sim 6 \text{ GeV}$ and does not fit the data points above 6 GeV . This is simply due to the two-body kinematics. In $K^+ \rightarrow \mu^+ \nu_\mu$ decay, the maximum energy of ν_μ is

$$E_\nu^{\text{max}} = \frac{m_K^2 - m_\mu^2}{2m_K \sin \theta} \quad (7)$$

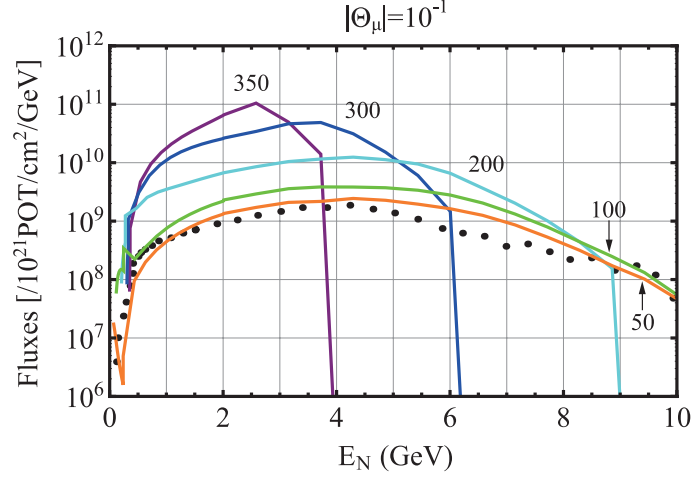


Figure 4: Fluxes of the heavy neutrino $\phi_N(E_N)$ from the $K^+ \rightarrow \mu^+ N$ mode for several sample values of M_N . Black dots shows $|\Theta_\mu|^2 \phi_{\nu_\mu}$. The masses are taken as $M_N = 350$ MeV (purple), 300 MeV (blue), 200 MeV (cyan), 100 MeV (green), 50 (orange) MeV.

for $\theta < 90^\circ$. The larger the off-axis, the smaller the maximum neutrino energy. The abundance of ν_μ above 6 GeV means that certain fraction of kaons must have momentum directing to the near detector so that ν_μ with smaller θ can contribute to the flux. This is of course expected since the magnetic horns do not make the kaon momenta perfectly parallel to the beam axis. The smaller angle $\theta_0 = 1.48^\circ$ effectively takes this effect into account.

3.2 The heavy neutrino flux

With the kaon spectrum $\phi_K(p_K)$ discussed above, let us estimate the heavy neutrino flux $\phi_N(E_N)$. The calculation goes as before. The source term is given by

$$S_N(E_N, \theta, \phi, l) = \int_0^\infty dp_K \phi_K(p_K, l) \left(\frac{m_K}{p_K} \right) \sum_{i=1}^2 \frac{d^3 \Gamma_i}{dE_N d\cos \theta d\phi}, \quad (8)$$

where Γ_1 and Γ_2 are the decay width for $K^+ \rightarrow \mu^+ N$ and $K^+ \rightarrow e^+ N$ respectively. Provided that the decay length of the heavy neutrino is much larger than the distance between the K^+ decay point and the detector, the heavy neutrino flux $\phi_N(E_N)$ is given by just replacing S_ν with S_N in Eq. (6). We will see in Section 4 that this is the case for the energy and the parameters of the current interests. Details on the differential decay rates $d\Gamma_i$ are shown in Appendix A.

Fig. 4 and 5 present the heavy neutrino fluxes $\phi_N(E_N)$ for several fixed values of M_N .

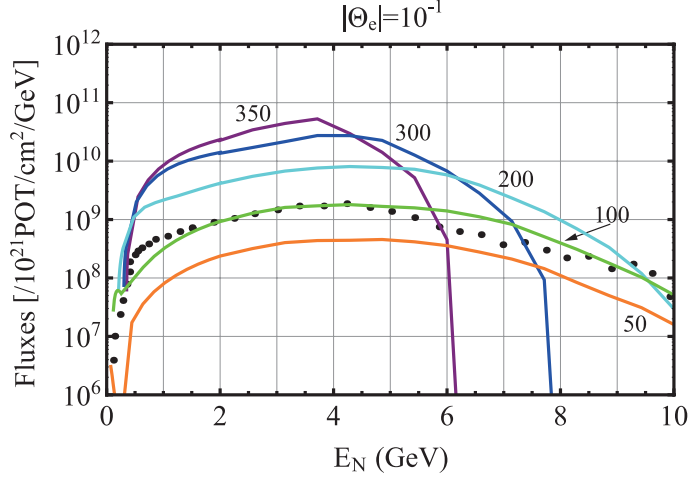


Figure 5: Same as Fig. 4 but for the $K^+ \rightarrow e^+ N$ mode.

Fig. 4 and 5 show the case where $(|\Theta_e|, |\Theta_\mu|) = (0, 10^{-1})$ and $(|\Theta_e|, |\Theta_\mu|) = (10^{-1}, 0)$ respectively. The masses are sampled as $M_N = 350$ MeV (purple), 300 MeV (blue), 200 MeV (cyan), 100 MeV (green), 50 (orange) MeV. In the both cases of $K^+ \rightarrow \mu^+ N$ and $K^+ \rightarrow e^+ N$, the spectral shapes are similar to ϕ_{ν_μ} for $M_N \lesssim 100$ MeV, whereas they are significantly deviated from ϕ_{ν_μ} for $M_N \gtrsim 100$ MeV. The most remarkable feature is the enhancement of the flux at lower energies for larger M_N . In Fig. 4, for example, the heavy neutrinos of $M_N = 350$ MeV gather around 2 – 3 GeV and the peak intensity reaches $10^{11} \text{ cm}^{-2} \text{ GeV}^{-1}$. This is nearly two orders of magnitude larger than the naive expectation $|\Theta_\mu|^2 \phi_{\nu_\mu}$.

There are two reasons for the enhancement. One is the spin conservation in the K^+ rest frame. The matrix elements of the decay processes $K^+ \rightarrow \mu^+ N$ and $K^+ \rightarrow e^+ N$ scale with M_N^2 when M_N is much larger than the mass of the charged lepton in each final state. This accounts for the larger fluxes than $|\Theta_{e,\mu}|^2 \phi_{\nu_\mu}$ for $M_N > m_\mu$. This also accounts for the smaller flux than $|\Theta_e|^2 \phi_{\nu_\mu}$ for $M_N = 50$ MeV in Fig. 5.

The another reason is slower motions of the heavy neutrinos at the rest frame of the parent particle. The smaller the daughter velocities, the easier to boost them into the forward directions. In the current setup, the detector may well be regarded as the object placed in the forward direction of the K^+ momenta. In the case of $K^+ \rightarrow \mu^+ \nu_\mu$, neutrinos can be emitted not only to the forward directions but also to the backward directions since the neutrino masses are very small and the neutrino velocities at the rest frame are larger than the parent typical velocity at the laboratory frame. On the other hand, in the decay

$K^+ \rightarrow \mu^+ N$, the heavy neutrinos tend to be emitted to the forward directions. In the rest frame of K^+ , the gamma factor of N at the K^+ rest frame is given by

$$\gamma_N = \frac{m_K^2 - m_\mu^2 + M_N^2}{2m_K M_N}. \quad (9)$$

This goes to unity as M_N approaches to the threshold value $M_N = m_K - m_\mu = 388 \text{ MeV}$. On the other hand, most of the kaons carry the momentum around $1 - 4 \text{ GeV}$, so that the gamma factor of K^+ is typically given by $\gamma_K = 2 - 8$. Then Eq. (9) tells us that $\gamma_K > \gamma_N$ for $M_N \gtrsim 120 \text{ MeV}$. Hence for $M_N \gtrsim 120 \text{ MeV}$, kaon's velocities overcome N 's velocities and N are focused into the forward directions. This agrees with the flux behavior seen in Fig. 4 and 5.

4 Event rates and expected sensitivity

As we have seen in Section 2, a fraction of the heavy neutrinos passing through the detector decay inside the detector volume and leave signals via various decay modes. In this section, we calculate the number of signal events in ND280 and estimate the potential sensitivity of ND280. We argue that the sensitivity of ND280 is comparable to that of the PS191 experiment.

4.1 Number of the signal events

The total number of events is given by the difference between the number of the heavy neutrinos at the up and down stream end of the detector. For a particular decay channel, the number of events is given by

$$\text{Events} = A \int_{M_N}^{\infty} dE_N \frac{1}{\lambda} \int_{x_0}^{x_1} dx \phi_N(E_N, x), \quad (10)$$

where λ is the (partial) decay length for the signal decay mode of interest, A is the cross-sectional area of the detector, x is the flight distance of the heavy neutrinos and (x_0, x_1) means the detector segment. The number of the heavy neutrinos decreases by the decays. With the total decay length Λ_N , the x dependence of $\phi_N(E_N, x)$ is determined as

$$\phi_N(E_N, x) = \phi_N(E_N) e^{-\frac{x}{\Lambda_N}}, \quad (11)$$

where $\phi_N(E_N)$ is the heavy neutrino flux discussed in Section 3.2.

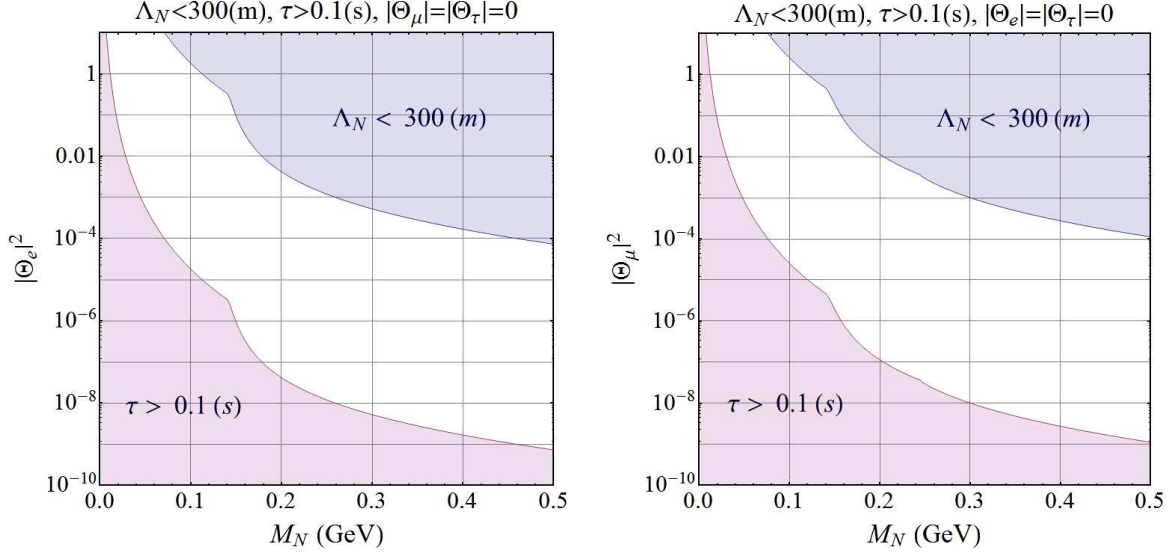


Figure 6: Parameter region that satisfy $\Lambda_N < 300$ m for $\gamma_N = \sqrt{2}$ (The filled region labeled by $\Lambda_N < 300$ (m)). The filled region labeled by $\tau > 0.1$ (s) shows the region where heavy neutrino decay may spoil BBN.

Eq. (10) is further simplified if the total decay length is much larger than the flight distance of the heavy neutrino. Provided that $\Lambda_N \gg x_0, x_1 - x_0$, the number of events reads

$$\text{Events} \simeq \int_{M_N}^{\infty} dE_N \phi_N(E_N) \frac{V}{\lambda}, \quad (12)$$

where $V = A(x_1 - x_0)$ is the detector volume. In the T2K setup, $x_0 \approx 300$ m and $x_1 - x_0 \approx 5$ m. Thus the condition $\Lambda_N \gg x_1 - x_0$ holds if $\Lambda_N \gg x_0 \approx 300$ m.

It turns out that the condition $\Lambda_N \gg 300$ m holds in the most parameter and energy region of interest. Fig. 6 highlights a “strong coupling” regime where $\Lambda_N < 300$ m. Here the region is showing an example with $\gamma_N = \sqrt{2}$ ($p_N = M_N$). For $\gamma_N = \sqrt{1.5}$ ($p_N = M_N/2$), the boundary is pushed down by factor of two. For the energies $\gamma_N \gtrsim \sqrt{2}$, the total decay is effective only for strong coupling regime $|\Theta_{e,\mu}|^2 \gtrsim 10^{-4}$, which is already ruled out by many experiments. Eq. (12) is well applicable for the following discussions which treat the region far below the bound shown in Fig. 6.

In Fig. 6, we put in passing the region where the lifetime of the heavy neutrino becomes long enough so that late time decay of the heavy neutrinos may spoil the success of Big Bang Nucleosynthesis (BBN). Ref. [31, 32] has studied such a bound for $10 \text{ MeV} < M_N < 140 \text{ MeV}$ in detail. For $M_N > 140 \text{ MeV}$, however, there is no consensus about the constraint from BBN. Here we simply present the region for $\tau > 0.1$ s [32] where the heavy

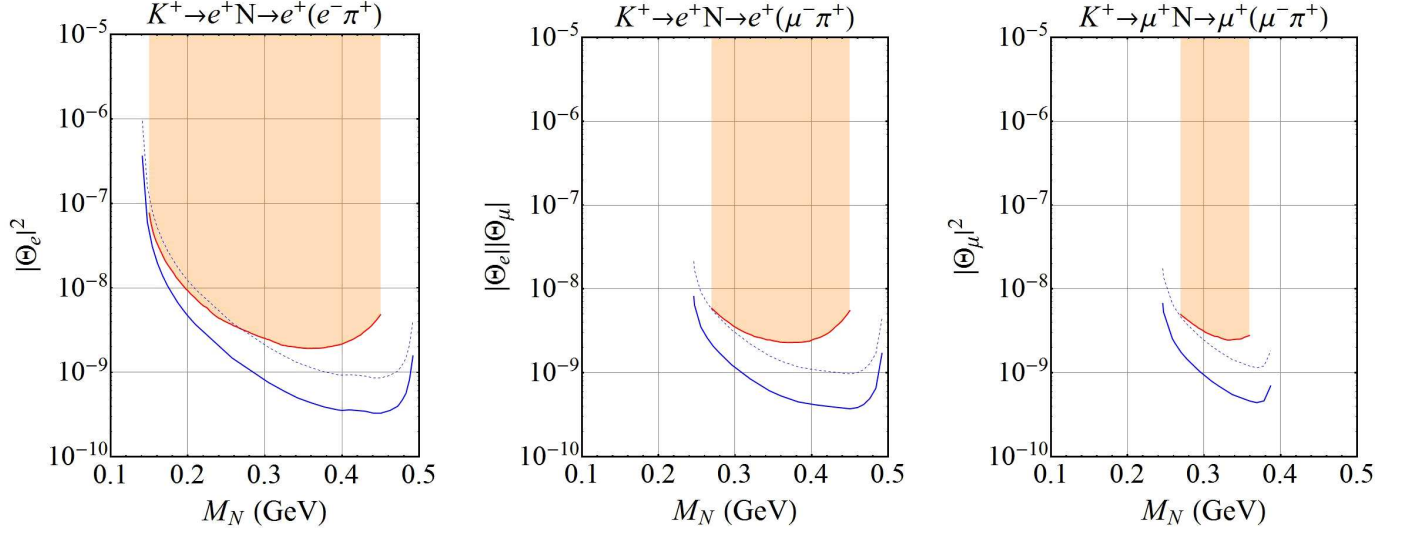


Figure 7: Expected sensitivity of T2K and upper limits by PS191. The blue solid curves show the contour for 2.44 events at T2K. The red curves show 90% CL upper bound by PS191 [19].

neutrinos are not cleared away before the onset of BBN.

4.2 Comparison between T2K and PS191

For each mass eigenstate of the heavy neutrinos, four additional parameters are introduced into the Standard Model; M_N and $\Theta_{e,\mu,\tau}$. Experiments impose some constraints on the four dimensional space $(M_N, |\Theta_e|, |\Theta_\mu|, |\Theta_\tau|)$. However, the analysis of experimental constraints on the full four-dimensional space is a complicated task. PS191 has made the following assumptions/simplifications in their analysis.

- Heavy neutrinos are Dirac particles.
- The NC contributions to the three-body decays of N are neglected.
- Either $K^+ \rightarrow \mu^- N$ or $K^+ \rightarrow e^- N$ is dominant in the production.

In the following, we first make the same simplifications, just aiming for a comparison between T2K and PS191. In Section 4.3, we give a comment on the case where the second simplification is relaxed so that the three-body decay depends on Θ_τ .

4.2.1 Two body channels

Let us first focus on the two body decays. Fig. 7 shows the expected sensitivity for the chains $K^+ \rightarrow e^+ N \rightarrow e^+ (e^- \pi^+)$ (left), $K^+ \rightarrow e^+ N \rightarrow e^+ (\mu^- \pi^+)$ (middle) and

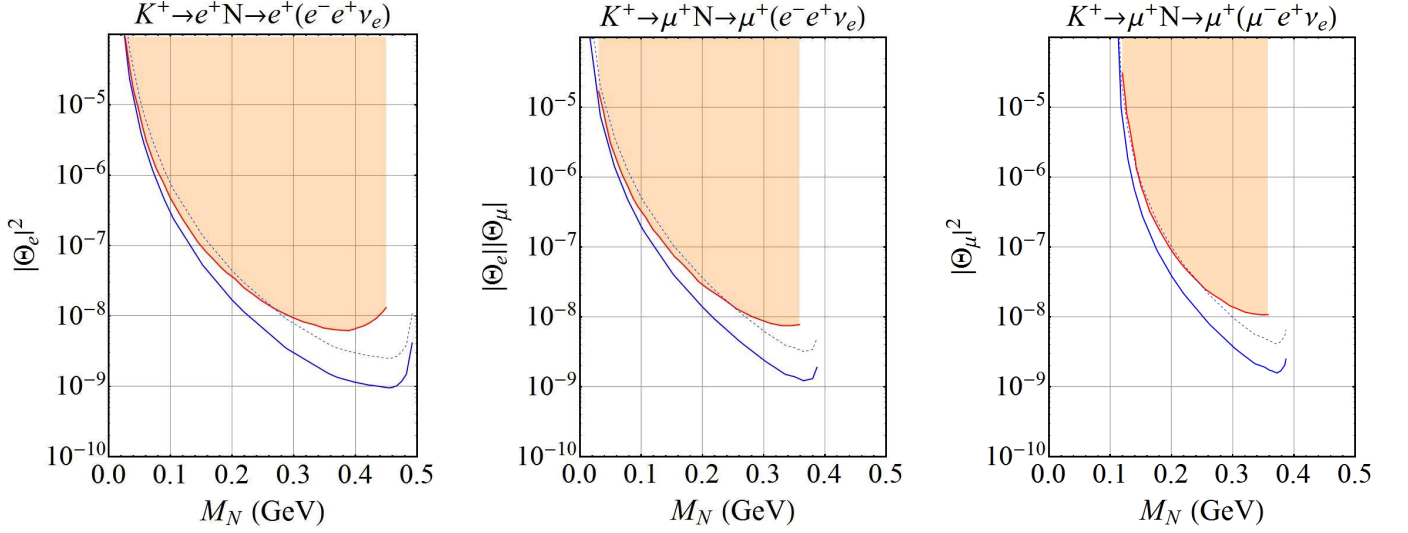


Figure 8: Same as Fig. 7 but for the three-body decay channels.

$K^+ \rightarrow \mu^+ N \rightarrow \mu^+ (\mu^- \pi^+)$ (right), respectively. In the figures, the red curves show 90% CL upper bound by PS191 [19]. The blue solid curves show the contour for 2.44 events, which corresponds to 90% CL limit when the measured signal and the expected background are null [33]. Here the fiducial volume of the detector is taken as $V = 3.5 \times 3.5 \times 5.0 = 61.25 \text{ m}^3$ [34].

The interactions between the active neutrinos and the nuclei in the detector provide backgrounds for the decay signals $N \rightarrow \mu^- \pi^+$ and $N \rightarrow e^- \pi^+$. For instance, the reactions

$$\begin{aligned} \nu_\mu + n &\rightarrow \mu^- + \pi^+ + n & (\text{CC} - n\pi^+) \\ \nu_\mu + {}^{16}\text{O} &\rightarrow \mu^- + \pi^+ + {}^{16}\text{O} & (\text{CC} - \text{coherent } \pi^+) \end{aligned}$$

may become background for $N \rightarrow \mu^- \pi^+$. It is expected that these events account for 4% of the whole neutrino events in ND280, resulting 7300 events/ 10^{21} POT/ton [34]. However the background can be reduced by taking the invariant mass of μ^- and π^+ momenta in the final state. For the heavy neutrino signal, the event distribution sharply peaks at the heavy neutrino mass while the ν_μ events provide continuous background. A serious sensitivity should be estimated together with the invariant mass distribution for the ν_μ reactions, the energy resolutions, all sort of uncertainties, etc. Such a thorough analysis is interesting but beyond the scope of this work.

We can further reduce the background by selecting the events taking place in the TPC volume which is filled by argon gas. Due to low density of the gas region, the ν_μ events are significantly reduced while keeping the signal rates unchanged. Out of the full volume of

61.25 m³, 9.0 m³ is filled with the argon gas [34] and available for this purpose. According to Ref. [35], the total neutrino events taking place in the argon gas are about 2000 at 10²¹ POT. Since 4% of them become the background, the number of the background event is expected to be around 80. These 80 events will be further reduced in the bin around the heavy neutrino mass. Keeping this in mind, we plot the contour for 2.44 events with $V = 9.0 \text{ m}^3$ by the dashed curves.

For $N \rightarrow e^- \pi^+$, the background processes are produced by the CC interactions of ν_e . However, the ν_e flux is about two orders of magnitude smaller than that of ν_μ around the peak energy $\sim 600 \text{ MeV}$ [27, 28, 29]. By selecting the events in the gas volume, the background rate is expected to be less than one for 10²¹ POT. The decay channel $N \rightarrow e^- \pi^+$ is more promising than $N \rightarrow \mu^- \pi^+$ in view of signal/background ratio if $|\Theta_e| \sim |\Theta_\mu|$.

4.2.2 Three body channels

Fig. 8 presents the same plots as Fig. 7 but for the three-body channels; $K^+ \rightarrow e^+ N \rightarrow e^+(e^- e^+ \nu_e)$ (left), $K^+ \rightarrow \mu^+ N \rightarrow \mu^+(e^- e^+ \nu_e)$ (middle), $K^+ \rightarrow \mu^+ N \rightarrow \mu^+(\mu^- e^+ \nu_e)$ (right).

A major obstacle to successful identification of $N \rightarrow e^- e^+ \nu$ would be π^0 that is copiously produced by neutrino interactions. The two photons from π^0 decay develop to electromagnetic cascades in the detector material and may mimic the signals. The subdominant decay mode $\pi^0 \rightarrow e^- e^+ \gamma$ (1.17%) may also contribute to the background when one of the final-state particle is undetected. The invariant mass distribution of the electron pair is useful since it moderately peaks at one-half of the heavy neutrino mass [21]. The analysis needs anyway precise understanding of the background, and the detection via $N \rightarrow e^- e^+ \nu$ seems less promising than the two-body modes.

As for $N \rightarrow \mu^- e^+ \nu$, the charmed-meson production by the neutrino CC interaction [36] and successive semi-leptonic decay may become the background. According to Ref. [36], the cross section of the charm production is about 1% (4%) of the total CC cross section for $E_\nu = 5 \text{ GeV}$ (15 GeV). Due to the off-axis technic, however, the contributions from such high-energy neutrinos are suppressed in the T2K setup. By selecting the events in argon gas, the neutrino reduction rates can be further reduced, and $N \rightarrow \mu^- e^+ \nu$ may become more or less background free.

Although PS191 has not studied the signal process $N \rightarrow \mu^- \mu^+ \nu$ open for $M_N >$

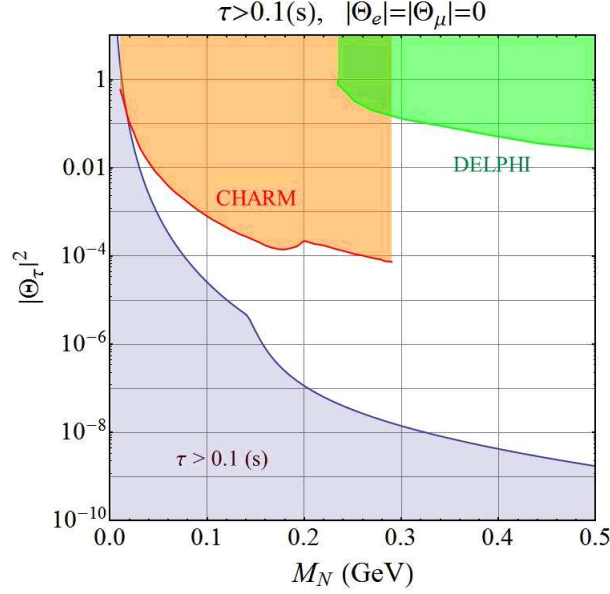


Figure 9: Allowed region of M_N - $|\Theta_\tau|^2$ plane. The upper-filled regions are excluded by CHARM [37] and DELPHI [38] at 90% and 95% CL, respectively. The lower-filled region is the regime where the lifetime of the heavy neutrino is longer than 0.1 s.

211 MeV, it should be emphasized that searching for this di-muon signal is also a promising method. The main background for $N \rightarrow \mu^- \mu^+ \nu$ may be the charmed-meson production by the neutrino CC interaction [36] and successive semi-leptonic decay as in the case of $N \rightarrow \mu^- e^+ \nu$. This rate is however expected to be small for the neutrino energy in T2K.

4.3 Implications of $\Theta_\tau \neq 0$

So far we have focused on the comparison between T2K and PS191 and Θ_τ is accordingly neglected. In this subsection, we comment on several implications of the $\Theta_\tau \neq 0$ case.

Since Θ_τ is not involved in the main production processes such as pion and kaon decays, the experimental constraints of $|\Theta_\tau|^2$ are much weaker than that of $|\Theta_e|^2$ and $|\Theta_\mu|^2$. Fig. 9 shows the allowed region of M_N - $|\Theta_\tau|^2$ plane. The upper-filled regions are excluded by CHARM [37] and DELPHI [38] at 90% and 95% CL, respectively. The lower-filled region is the regime where the lifetime of the heavy neutrino is longer than 0.1 s. By comparing Fig. 9 and Fig. 6, it is seen that, unlike $|\Theta_{e,\mu}|^2$, $|\Theta_\tau|^2$ can be large enough so that the BBN constraint is avoided without contradicting with the upper bound from the direct experiments.

While Θ_τ is not involved in the production processes, the detection processes in general

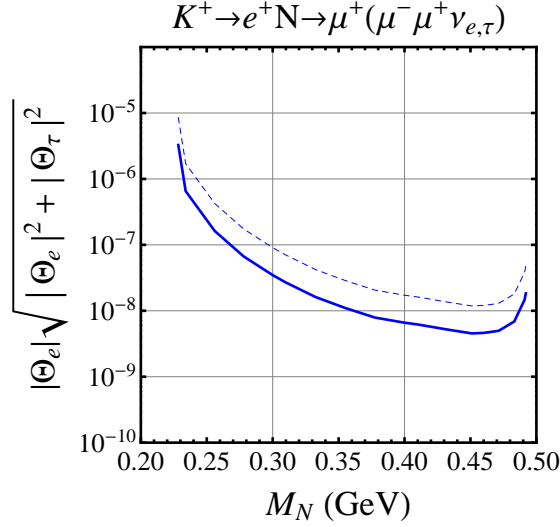


Figure 10: Expected sensitivity for $N \rightarrow \mu^- \mu^+ \nu$. $|\Theta_\mu| = 0$ is assumed.

depend on Θ_τ since $N \rightarrow e^- e^+ \nu$ and $N \rightarrow \mu^- \mu^+ \nu$ are induced not only by the CC but also by the NC interactions [20]. This means the experimental setup discussed in this paper has sensitivities to certain combinations of $|\Theta_{e,\mu}|^2$ and $|\Theta_\tau|^2$. To demonstrate this, let us focus on a simple case where $|\Theta_e|^2, |\Theta_\tau|^2 \gg |\Theta_\mu|^2$ and $M_N > 211$ MeV. In this case, the heavy neutrinos are produced by $K^+ \rightarrow e^+ N$ and can be detected by $N \rightarrow \mu^- \mu^+ \nu$. Since $|\Theta_\mu|^2$ is small, the decay process $N \rightarrow \mu^- \mu^+ \nu$ is conducted only by the NC interactions and the decay width becomes proportional to $|\Theta_e|^2 + |\Theta_\tau|^2$. Therefore by analyzing the dimuon signals in the detector, one is able to constraint $|\Theta_e| \sqrt{|\Theta_e|^2 + |\Theta_\tau|^2}$ for $M_N > 211$ MeV (or discover the heavy neutrino).

Fig. 10 shows the expected sensitivity for $N \rightarrow \mu^- \mu^+ \nu$. As in Fig. 7 and Fig. 8, the solid (dashed) curve is the contour for 2.44 events with $V = 61.25$ (9.0) m^3 . From Fig. 10 and Fig. 9, one can see that there exists the parameter regime where the dimuon signal is sizable while the success of BBN is unspoiled. For example, it is seen that the combination $|\Theta_e| = 10^{-4.5}$ and $|\Theta_\tau|^{-2.5}$ is BBN safe but $\mathcal{O}(10) - \mathcal{O}(10^2)$ events of $N \rightarrow \mu^- \mu^+ \nu$ are expected for $300 \text{ MeV} \lesssim M_N \lesssim 400 \text{ MeV}$. Interestingly, this case also predicts $\mathcal{O}(1) - \mathcal{O}(10)$ events of $N \rightarrow e^- \pi^+$ (see the left pannel of Fig. 7), so that the heavy neutrino model can be tested in multi-dimensional way.

5 Conclusions

In this paper, we have focused on the heavy (sterile) neutrinos produced by kaon decays and explored the feasibility of their detection at the existing facilities of the accelerator-based neutrino experiment. Taking the T2K experiment as a typical example, we have estimated the heavy neutrino fluxes produced in the beam line and calculated the event rates of their decay taking place inside the detector.

Due to massive nature of the heavy neutrino, the spectrum of the heavy neutrino is significantly different from that of the ordinary neutrinos. The ordinary neutrinos are emitted to various directions in the laboratory frame due to their tiny masses. On the other hand the heavy neutrinos carrying a large mass tend to be emitted to the forward directions and frequently hit the detector. This is a unique advantage of the experiments in which the parent mesons decay in flight with sufficient gamma factors.

Among various decay modes, $N \rightarrow e^- \pi^+$ open for $M_N > 140 \text{ MeV}$ is one of the most promising channels for detection because of its larger rate and lower background. The backgrounds from the active neutrino reactions can be reduced by selecting the events occurring in the regions filled with no material. In the T2K near detector, the TPC volume 9 m^3 filled with argon gas plays this role. The expected sensitivity for this mode is comparable to that of PS191, which has placed the most stringent bound on the heavy neutrino mixing.

The three body modes $N \rightarrow e^- e^+ \nu$, $N \rightarrow \mu^- e^+ \nu$, and $N \rightarrow \mu^- \mu^+ \nu$ are also interesting signals to search for. In particular, $N \rightarrow e^- e^+ \nu$ and $N \rightarrow \mu^- \mu^+ \nu$ are conducted not only by the charged current but also the neutral current, so that the tau flavor mixing Θ_τ is involved in the detection probabilities. Since $|\Theta_\tau|$ is less constrained than $|\Theta_e|$ and $|\Theta_\mu|$, the above two modes are not necessarily suppressed when $|\Theta_e|$ and $|\Theta_\mu|$ are small such that the two-body modes $N \rightarrow e^- \pi^+$ and $N \rightarrow \mu^- \pi^+$ are beyond the reach.

Finally, we would like to emphasize that two quasi-degenerate heavy neutrinos of $\mathcal{O}(100) \text{ MeV} - \mathcal{O}(10) \text{ GeV}$ can account for not only the neutrino masses in oscillation experiments but also the baryon asymmetry of the universe [4, 5, 6]. The heavy neutrinos studied in this work are thus quite interesting targets to search for. Furthermore, Ref. [7] reports that the sterile neutrinos with mass $\sim 200 \text{ MeV}$ could facilitate the energy transport from the supernova core to the shock front, prompting a successful explosion. The needed mixing is either $|\Theta_\tau|^2 > 10^{-8}$ or $10^{-7} - 10^{-8}$ for $|\Theta_\mu|^2$. Interestingly, T2K can probe latter case via the $N \rightarrow \mu^- e^+ \nu$ mode (see the right panel of Fig. 8). In addition,

heavy neutrinos with masses smaller than $\mathcal{O}(100)$ MeV may give a significant effect on the neutrinoless double beta decays [39]. According to a rough estimation in Table 1, MiniBooNE and SciBooNE have comparable abilities to T2K so that these experiments equally have the chance to probe these interesting possibilities. Serious analyses by these collaborations may lead to the discovery of the heavy neutrinos and revolutionize neutrino physics.

Acknowledgments

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A Details on the flux calculation

In this appendix, we present technical details on the flux calculation discussed in Section 3. Let us start from the source term Eq. (4),

$$S_\nu(E_\nu, \theta, \phi, l) = \int_0^\infty dp_K \phi_K(p_K, l) \left(\frac{m_K}{p_K} \right) \frac{d^3\Gamma}{dE_\nu d\cos\theta d\phi}. \quad (13)$$

As discussed in Section 3, the neutrino flux $\phi_{\nu_\mu}(E_\nu)$ of Eq. (6) is obtained by integrating this over θ, ϕ, l . The computation is greatly simplified if the kaon momentum is parallel to the beam axis. The detector geometry for this case is presented in Fig. 11. Under this simplification, the neutrino flux follows

$$\phi_{\nu_\mu}(E_\nu) = \frac{\Delta\phi}{A} \int_0^{l_f} dl \int_{-1}^1 d\cos\theta S_\nu(E_\nu, \theta, l) P'(\theta, \theta_0), \quad (14)$$

where $\Delta\phi = r/R \sin\theta_0$ and $l_f = 96$ m. The function $P'(\theta, \theta_0)$ projects out the general polar angles into the ones with which ν_μ pass through the near detector. It is given by

$$P'(\theta, \theta_0) = H(\theta - \theta^-) H(\theta^+ - \theta), \quad (15)$$

where $H(x)$ is the Heaviside step function and

$$\theta^- = \theta(l) - \frac{r}{2R(l)}, \quad \theta^+ = \theta(l) + \frac{r}{2R(l)}, \quad (16)$$

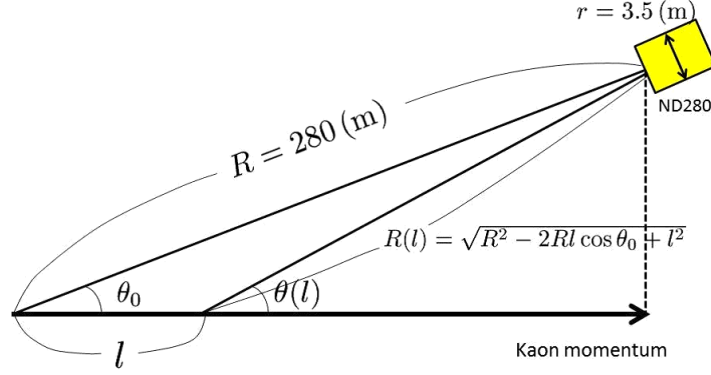


Figure 11: Geometry of the near detector ND280 and the kaon momentum.

$$\theta(l) = \arcsin \left[\frac{R \sin \theta_0}{R(l)} \right], \quad R(l) = \sqrt{R^2 - 2Rl \cos \theta_0 + l^2}, \quad (17)$$

with $R = 280$ m and $r = 3.5$ m.

The K^+ spectrum $d\sigma/dp_K$ in the proton collision with a graphite target is measured by NA61/SHINE Collaboration [30]. Fig. 12 shows the best-fit values of the data [30] and an example of the fit. The spectrum is the average over the K^+ polar angles (relative to the beam axis) $20 - 140$ mrad where the probability that daughter neutrinos pass through ND280 is high [29, 30]. We assume that the shape of the kaon spectrum $\phi_K(p_K)$ is not far from this measured spectrum and use the following fitting;

$$\phi_K(p_K) = a_0 \left(\frac{d\sigma'}{dp_K} \right),$$

$$\frac{d\sigma'}{dp_K} = \frac{\phi_L \phi_H}{\phi_L + \phi_H}, \quad \phi_L = a_L p_K^{b_L}, \quad \phi_H = a_H (p_K + p_0)^{-b_H}, \quad (18)$$

where $a_L = 1.285$ mb GeV $^{-(b_1+1)}$, $b_L = 0.6289$, $a_H = 793.5$ mb GeV $^{-(b_2+1)}$, $b_H = 3.230$, with which $d\sigma'/dp_K = d\sigma/dp_K$ at $p_0 = 0$. A positive value of p_0 shifts the peak of $d\sigma/dp_K$ to lower energies.

The differential decay width in Eq. (13) is given by

$$\frac{d^3\Gamma}{dE_\nu d\cos\theta d\phi} = \frac{M^2}{2E_K} \frac{E_\nu}{8\pi^2} \delta[f(p_K)], \quad (19)$$

where

$$M^2 \equiv 2G_F^2 f_K^2 m_K^4 |V_{us}|^2 \left[\left(\frac{m_\mu}{m_K} \right)^2 - \left(\frac{m_\mu}{m_K} \right)^4 \right], \quad (20)$$

$$f(p_K) = m_K^2 - m_\mu^2 - 2E_\nu \sqrt{p_K^2 + m_K^2} + 2p_K E_\nu \cos\theta. \quad (21)$$

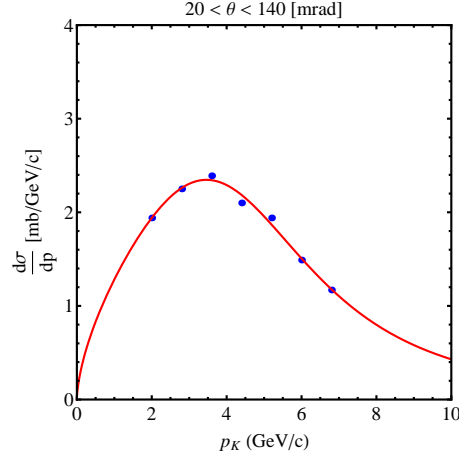


Figure 12: The K^+ spectrum $d\sigma/dp_K$ in proton-carbon collisions at 31 GeV proton energy. The dots are the best-fit values of the data [30] and the solid line is the fit.

An explicit expression of Eq. (14) is written as

$$\phi_{\nu_\mu}(E_\nu) = \frac{\Delta\phi}{A} \int_0^{l_f} dl \int_{-1}^1 d\cos\theta F_{\text{low}} P'(\theta, \theta_0), \quad (22)$$

for $0 < E_\nu \leq (m_K^2 - m_\mu^2)/2m_K$ and

$$\phi_{\nu_\mu}(E_\nu) = \frac{\Delta\phi}{A} \int_0^{l_f} dl \int_{\cos\theta_c}^1 d\cos\theta F_{\text{high}} P'(\theta, \theta_0) \quad (23)$$

for $E_\nu \geq (m_K^2 - m_\mu^2)/2m_K$. Here

$$F_{\text{low}} = \frac{M^2}{8\pi^2} \left[\phi(p_K^+, l) \left(\frac{m_K}{p_K^+} \right) \frac{1}{2E_K(p_K^+)} \frac{1}{2 \left| \cos\theta - \frac{p_K^+}{E_K(p_K^+)} \right|} \right], \quad (24)$$

$$F_{\text{high}} = \frac{M^2}{8\pi^2} \left[\phi(p_K^-, l) \left(\frac{m_K}{p_K^-} \right) \frac{1}{2E_K(p_K^-)} \frac{1}{2 \left| \cos\theta - \frac{p_K^-}{E_K(p_K^-)} \right|} \right. \\ \left. + \phi(p_K^+, l) \left(\frac{m_K}{p_K^+} \right) \frac{1}{2E_K(p_K^+)} \frac{1}{2 \left| \cos\theta - \frac{p_K^+}{E_K(p_K^+)} \right|} \right], \quad (25)$$

$$p_K^\pm = \frac{(m_K^2 - m_\mu^2) \cos\theta \pm \sqrt{(m_K^2 - m_\mu^2)^2 - 4(1 - \cos^2\theta)m_K^2 E_\nu^2}}{2(1 - \cos^2\theta)E_\nu}, \quad (26)$$

$$\theta_c = \arcsin \left[\frac{m_K^2 - m_\mu^2}{2m_K E_\nu} \right]. \quad (27)$$

For the heavy neutrino production $K^+ \rightarrow \mu^+ N$, the differential decay width is given by

$$\frac{d^3\Gamma_1}{dE_N d\cos\theta d\phi} = \frac{|\Theta_\mu|^2 M_1^2}{2E_K} \frac{p_N}{8\pi^2} \delta[g(p_K)], \quad (28)$$

where

$$M_1^2 \equiv 2G_F^2 f_K^2 m_K^4 |V_{us}|^2 \left[\left(\frac{M_N}{m_K} \right)^2 + \left(\frac{m_\mu}{m_K} \right)^2 - \left(\left(\frac{M_N}{m_K} \right)^2 - \left(\frac{m_\mu}{m_K} \right)^2 \right)^2 \right], \quad (29)$$

$$g(p_K) = m_K^2 + M_N^2 - m_\mu^2 - 2E_N \sqrt{p_K^2 + m_K^2} + 2p_K p_N \cos\theta. \quad (30)$$

The formula for $K^+ \rightarrow e^+ N$ is obtained by making the replacement $\mu \rightarrow e$ in the above. By replacing the differential decay width in Eq. (13) with Eq. (28), it is straightforward to obtain the formulas for $\phi_N(E_N)$ similar to Eq. (22) and Eq. (23).

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